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GIMP application

Subject: **Math (III grade)**

Topic: **Trigonometric functions**

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Topic: Trigonometric functions

Level: High school

Language: English

Type of material: Instructions for working with new topic

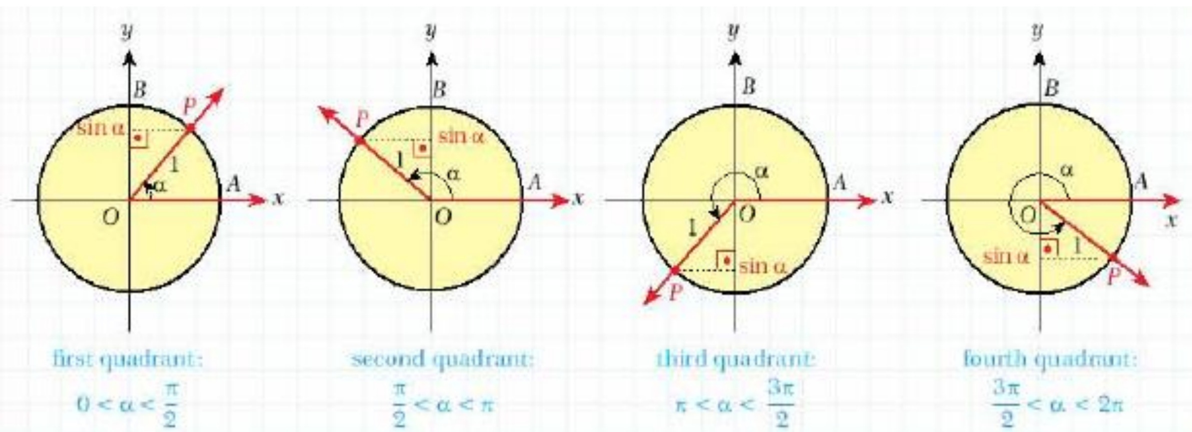
Material format: PDF document

Abstract: We can calculate trigonometric ratios for any angle. In this section we will extend our knowledge of trigonometric ratios to cover all angles. To do this, we will study the trigonometric function in the context of the unit circle. The trigonometric function are also called circular functions since they can be defined on the unit circle.

The Sine Function

Definition: Let |OP| be the terminal side of an angle α in standard position such that P lies on the unit circle. Then the ordinate (y-coordinate) of the point P on the unit circle is called *the sine of angle α*. It is denoted by sinα. The function which matches a real number α to the real number sinα is called *the sine function*.

The figures below show how the value of sinα changes as the point P moves round the unit circle. Since the sine value of α is the ordinate of the point P, the y-axis can also be called *the sine axis*.



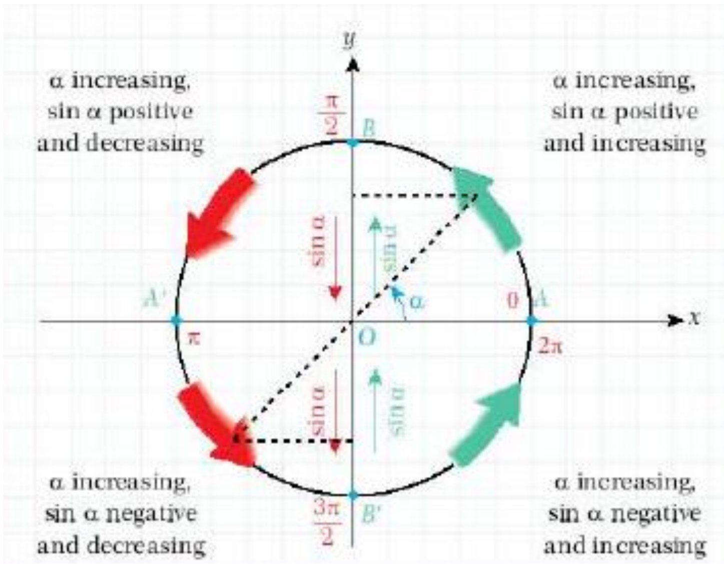
As we can see in the figures:

if $0 < \alpha < \frac{\pi}{2}$ then $0 < \sin\alpha < 1$;

if $\frac{\pi}{2} < \alpha < \pi$ then $0 < \sin\alpha < 1$;

if $\pi < \alpha < \frac{3\pi}{2}$ then $-1 < \sin\alpha < 0$;

if $\frac{3\pi}{2} < \alpha < 2\pi$ then $-1 < \sin\alpha < 0$.



Since sinα is the ordinate of the point P, we can also easily observe how it increases and decreases as P moves round the circle.

Conclusion:

1. For all values of α the sine function takes values between -1 and 1 i.e. $-1 \le \sin\alpha \le 1$.
2. As α increases, the sine function increases in the first and fourth quadrants and decreases in the second and third quadrants.
3. The sine function is positive in the first and the second quadrants and negative in the third and fourth quadrants.

Example 1: Simplify the expressions.

a. $2\cdot\sin\frac{3\pi}{2} - \sin\pi + 3\cdot\sin\frac{\pi}{2} - \sin 2\pi$

b. $-3\cdot\sin\frac{3\pi}{2} - 8\cdot\sin\pi + 5\cdot\sin\frac{\pi}{2} + \sin 0$

Solution:

a. $2\cdot(-1) - 0 + 3\cdot 1 - 0 = -2 + 3 = 1$

b. $-3\cdot(-1) - 8\cdot 0 + 5\cdot 1 + 0 = 3 + 0 + 5 = 8$

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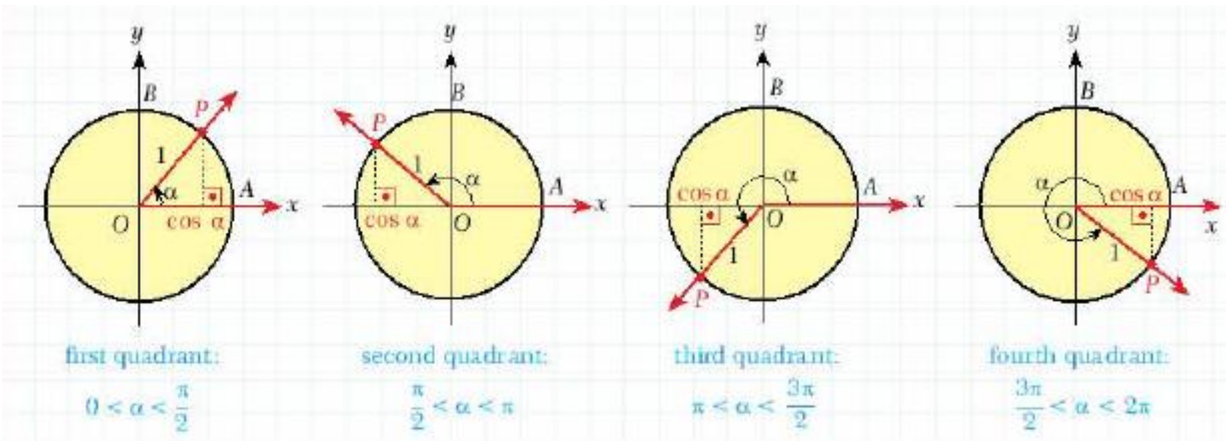
The Cosine Function

Definition: Let |OP| be the terminal side of an angle α in standard position such that P lies on the unit circle. Then the abscissa (x-coordinate) of the point P on the unit circle is called *the cosine of angle α*. It is denoted by cosα. The function which matches a real number α to the real number cosα is called *the cosine function*.

The figures below show how the value of cosα changes as the point P moves round the unit circle. Since the cosine value of α is the abscissa of the point P, the x-axis can also be called *the cosine axis*.



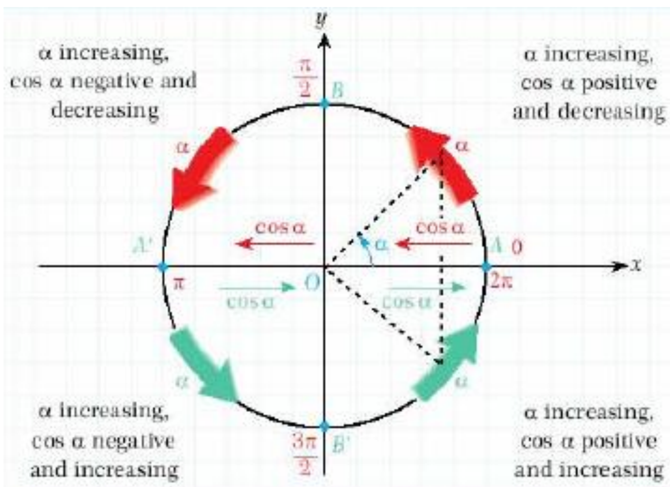
As we can see in the figures:

if $0<\alpha<\dfrac{\pi}{2}$ then $0<\cos\alpha<1$;

if $\dfrac{\pi}{2}<\alpha<\pi$ then $-1<\sin\alpha<0$;

if $\pi<\alpha<\dfrac{3\pi}{2}$ then $-1<\sin\alpha<0$;

if $\dfrac{3\pi}{2}<\alpha<2\pi$ then $0<\cos\alpha<1$.



Since cosα is the abscissa of the point P, we can also easily observe how it increases and decreases as P moves round the circle.

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Conclusion

1. For all values of α the cosine function takes values between -1 and 1 i.e. $-1 \le \cos\alpha \le 1$.
2. As α increases, the cosine function decreases in the first and second quadrants and increases in the third and fourth quadrants.
3. The cosine function is positive in the first and fourth quadrants and negative in the third and the second quadrants.

Notice that substituting the values y=sinθ and x=cosθ in the unit circle equation $x^2 + y^2 = 1$ gives us the relation $\cos^2\theta + \sin^2\theta = 1$.

Example 2: Find the minimum and maximum possible values of A in each expressions.

a. A=3cosx-1
b. A=2-4sinx

Solution:

a. We know that $-1 \le \cos x \le 1$, so

$$-3 \le 3\cos x \le 3$$

$$-3-1 \le 3\cos x - 1 \le 3-1$$

$$-4 \le 3\cos x - 1 \le 2$$

So min(A)=-4 and max(A)=2.

b. We know that $-1 \le \sin x \le 1$, so

$$-4 \le 4\sin x \le 4$$

$$4 \ge -4\sin x \ge -4.$$

We can rearrange this to get

$$-4 \le -4\sin x \le 4$$

$$-4+2 \le 2-4\sin x \le 4+2$$

$$-2 \le 2-4\sin x \le 6$$

So min(A)=-2 and max(A)=6.